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Gampaha Education Zone

දෙවන වාර ඇගයීම - 2025
Second Term Evaluation - 2025
இரண்டாம் தவணைப் பரீட்சை - 2025

ශ්‍රේණිය Grade **13**	විෂයය Subject **Combined Mathematics I**	කාලය Time **3h 10 min**
නම பெயர் Name		

Instructions:

- This paper consists of two parts.
  Part A (Questions 1-10) & **Part B (Questions 11-16)**
- **Part A**
  Answer **all** the questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- **Part B**
  Answer **five** questions only. Write your answers on the sheets provided.
- At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.

You are permitted to remove **only Part B** of the question paper from the examination hall.

For Examiners' Use Only.

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	**1**	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	**11**	
	12	
	13	
	14	
	15	
	16	
	Total	

Total

In words	
In numbers	

Code Numbers

Marking Examiner	
Checked by	
Superviced by	

Part A

1. Using the principle of Mathematical induction prove that $\sum_{r=1}^{n} 6r^2 - 2r - 1 = n(2n^2 + 2n - 1)$ for $n \in \mathbb{Z}^+$.

2. Let $y = |x - 2|$ and $y = 4 - |x|$. Sketch both graphs in the same diagram. Hence or otherwise, find the area bounded by the both graphs.

3. Using the ten digits from 0 to 9, it is necessary to form 6 digits secret codes of which the first two digits are 2 or 8 and without repeating any digit. How many such secret codes can be formed? How many of them are even numbers?

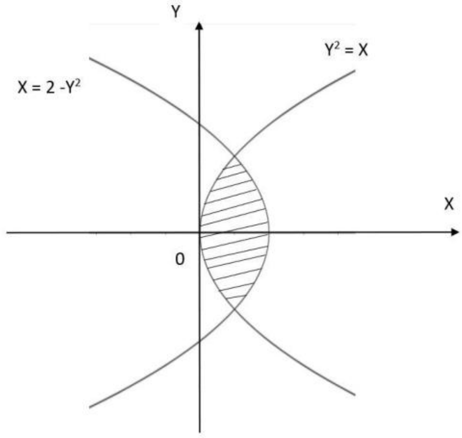
....

4. Find the real value of a such that $\lim_{x \to 0} \frac{[(a^3+x)^{1/3} - a]\sin 2x}{x^2} = \frac{1}{6}$. Where $a > 0$

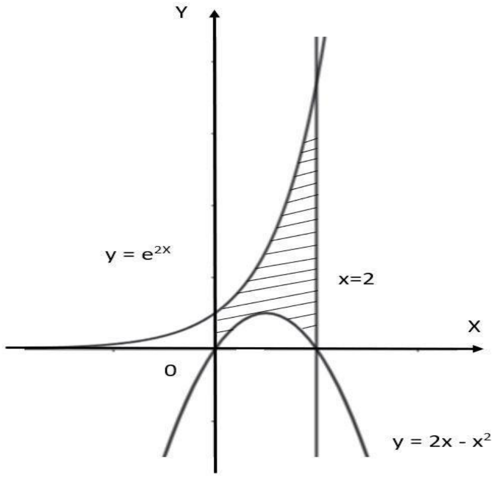
....

5. The parametric form of a curve is given by $x = sec^2 t$ and $y = \cos t$, where t is a parameter.
Show that the equation of the tangent drawn to the curve at the point P corresponding to $t = \frac{\pi}{4}$, is given by $x + 4\sqrt{2y} - 6 = 0$

6. Show that the area of the shaded region enclosed by the two symmetric graphs $y^2 = x$ and $x = 2 - y^2$ is $\dfrac{8}{3}$ sq. units.



7. The shaded area is enclosed by the two curves, $y = e^{2x}$, $y = 2x$and the straight lines $x = 0$ and $x = 2$. Show that the volume of the solid generate by rotating the shaded area about the x – axis in 2π radius is, $\frac{\pi}{60}(15e^{8} - 79)$ cubic units.



8. The equation of a side of a square is given by $x - 2y = 0$. Both diagonals meet at the point $\left(\frac{5}{2}, \frac{5}{2}\right)$.
Find the equations of both diagonals.

9. A circle which pass through the point $A(2,7)$, touches the y-axis at the point $B(0,3)$. Find the equation of this circle. Also show that the equation of the tangent drawn to tis circle at the point A is $3x-4y+22=0$.

10. If $\alpha+\theta=\frac{\pi}{6}$, then show that $\left(\sqrt{3}+\tan\theta\right)\left(\sqrt{3}+\tan\alpha\right)=4$. Deduce the value of $\tan\frac{\pi}{12}$.

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Part *B*

11. a). Let $f(x) \equiv x^2 + ax + b$ and $g(x) = x^2 + acx + bc^2$ for $a, b, c \in \mathbb{R}$ and $c \neq 1$.
If $\alpha$and β are real roots of the quadradic equation $f(x) = 0$, write $\alpha + \beta$ and $\alpha\,\beta$, in terms of a and b.

Then find the product of $(\alpha - \beta^2)(\alpha^2 - \beta)$, in terms of a and b.
If one root is square of other root of $f(x) = 0$, deduce that $a^3 + b^2 + b = 3ab$.

If λ and μ are the roots of the quadratic equation $g(x) = 0$, then show that the quadratic equation whose roots are $(\alpha\lambda + \beta\mu)$ and $(\alpha\mu + \beta\lambda)$ is given by $x^2 - a^2cx + 2c^2b(a^2 - 2b) = 0$.
If both $f(x) = 0$ *and* $g(x) = 0$, hold a common root, then show that $b(c+1)^2 = ca^2$.

b).Let $f(x) = x^4 + 2x^3 - 25x^2 - 26x + p$, when $p \in \mathbb{R}$.
If $(x - 2)$ is a factor of $f(x)$, find the volume of p.
For this value of p,express $f(x)$ in the form of $f(x) \equiv (x^2 + x + a)(x^2 + x + b)$., where a and b are constants to be determine.
Hence express $f(x)$ as a product of linear factors.
Solve the inequality $f(x) \geq 0$.
Deduce the remainder when the function $x^4 + 2x^3 - 25x^2 - 26x$ is divided by $(x^2 + 3x - 10)$.

12 . a). Let $u_r = \dfrac{4r^2+1}{4r^2-1}$ for $r \in \mathbb{Z}^+$.
Find a function $f(r)$ such that $u_r = 1 + f(r) - f(r+1)$ for $r \in \mathbb{Z}^+$.
Hence show that $\sum_{r=1}^{n} u_r = n + 1 - \dfrac{1}{2n+1}$.

Deduce that the series $\sum_{r=1}^{\infty} u_r$ is not convergent.

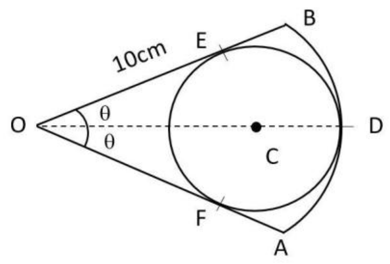
Is the series $\sum_{r=1}^{\infty} (u_r - 1)$ is convergent? Justify your answer.

b). A team of four students should be select for a competition, from a group of students consisting 3 male students and 2 female students from the Bio stream, 4 male students and 2 female students from Maths stream.
Find the number of different ways in which the team can be formed in each of the following cases.
i. If 2 male students and 2 female students in the team.
ii. At least 2 male students from the Maths stream and a female student from the Bio stream.

13. a). The figure shows a sector OAB of a circle of center O and radius 10 cm. $A\hat{O}B = 2\theta$ radians when $0 < \theta < \frac{\pi}{2}$.

This circle of Center C and radius r touches the AB at D.

OA and OB are two tangents to this circle at E and F respectively.



Write down OC in terms of r and show that $r = \dfrac{10\sin\theta}{1+\sin\theta}$, where θ is a variable.

Find $\dfrac{dr}{d\theta}$ for $r = \dfrac{10}{3}$.

If r is increasing at a rate of $2\ cms^{-1}$, find the rate at which θ is increasing when $\theta = \dfrac{\pi}{6}$.

b).Let $f(x) = \dfrac{ax+b}{(x+a)^2}$ $for\ x \neq (-a)$ and $a, b \in \mathbb{R}$.

It is given that the vertical asymptotes of the graph is $x = (-3)$. Also, the graph cuts the x-axis at $x = -\dfrac{2}{3}$. Find the values of a and b.

Show that $f'(x) = \dfrac{5-3x}{(x+a)^3}$ for this value of a. $where\ \ x \neq (-a)$

Find the interval on which the function $f(x)$ is increasing and the intervals on which the function $f(x)$is and decreasing.

Let $f''(x) = \dfrac{3(2x-8)}{(x+a)^4}$. Sketch the graph of $y = f(x)$ indicating turning points, asymptotes, axis intercepts and points of inflection.

Hence state the range of $f(x)$.

Find the value of k , such that $k \in \mathbb{R}^+$ of which the function $f(x)$ is one to one , on the interval $x \in [k, \infty)$.

14. a). Find the values $A, B, and\ C$ such that $1 \equiv A(x^2+8) + (Bx+C)(x+2)$; where $A, B, C\ \in \mathbb{R}$.

Hence evaluate $\displaystyle\int \frac{24}{(x+2)(x^2+8)}dx$

b). Using integration by parts evaluate $\displaystyle\int_0^1 x^2 e^{-x}dx$.

Hence find $\displaystyle\int_0^1 x^5 e^{-x^2}dx$, using a suitable substitution.

Find the real values of λ and μ such that the area bounded by the curve $y = 2x^5e^{-x^2}$ and the lines $x = 1, x = 0$ and $y = 0$ is $\lambda e^{-1} + \mu$.

c). Let $I_n = \displaystyle\int \frac{x^n}{\sqrt{x^2+5}}dx$

Show that $nI_n + 5(n-1)I_{n-2} = x^{n-1}\sqrt{5+x^2}$, for $n \geq 2$

Hence show that $\displaystyle\int_0^2 \frac{x^5}{\sqrt{x^2+5}}dx = \frac{168}{5} - \frac{40\sqrt{5}}{3}$

15. Prove that the perpendicular distance drawn to a line $px + qy + r = 0$ from a fixed point $A(\alpha, \beta)$ is given by $\dfrac{|p\alpha + q\beta + r|}{\sqrt{p^2 + q^2}}$, when $p, q, r, \alpha, \beta \in \mathbb{R}$

The perpendicular distance from the point $P(a, b)$ to each of the lines $y = 3x$ and $x = 3y$ is equal. Find a in terms of b , where $a, b \in \mathbb{R}$.

Given that a and b are both positive, deduce that the perpendicular distance to any of the above two lines from $P(a, b)$ is $\dfrac{a\sqrt{10}}{5}$.

Hence show that there exist two circles pass through the point $Q(2,1)$ and touching both lines $y = 3x$ and $x = 3y$.

Also show that the equation of one of the circles is given by $2x^2 + 2y^2 - 5x - 5y + 5 = 0$ *and* find the equation of the other circle.

Show also that these two circles intersect each other at two distinct points.

Find the equation of the common chord.

16. a) For $0 < \theta < 2\pi$, it is given that $\sin\theta + \cos\theta = \sin 2\theta + \cos 2\theta$. Show that $\tan\dfrac{3\theta}{2} = 1$.

Hence find all solutions of the equation $\sin 2\theta - \sin\theta + \cos 2\theta - \cos\theta = 0$.

b). State the Cosine rule for a triangle ABC, from the usual notation.

If a, b *and* c are successive terms of an arithmetic progression,

show that $\cos A + \cos C + 2\cos B = 2$

Deduce that $\cos\left(\dfrac{A - C}{2}\right) = 2\sin\dfrac{B}{2}$

c). Let $f(x) \equiv \cos 2x - \sqrt{3}[\cos x + \sin x]^2$

Find λ, μ *and* α when $\lambda, \mu \in \mathbb{R}$ and $0 < \alpha < \frac{\pi}{2}$ such that $f(x) \equiv \lambda\cos(2x + \alpha) + \mu$

Hence draw the graph of $y = f(x)$ in the range $-{}^{\pi}/_{4} \leq x \leq {}^{3\pi}/_{4}$.

d). Show that $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{12}{5} = \frac{\pi}{2}$

Deduce that $sin\left\{\frac{\pi}{4} - \frac{1}{2}tan^{-1}\frac{12}{5}\right\} = \frac{1}{\sqrt{26}}$